AP CALCULUS AB CRAM SHEET

2026 Exam | Every Formula, Theorem & Strategy You Need | College Board Aligned

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EXAM FORMAT & SCORING

Section	Questions	Time	Weight
I-A: MCQ No Calc	30 questions	60 min	50%
I-B: MCQ Calculator	15 questions	45 min	
II-A: FRQ Calculator	2 questions	30 min	50%
II-B: FRQ No Calc	4 questions	60 min	

Total: 45 MCQ + 6 FRQ = 3 hrs 15 min. Score 1-5. Roughly 60-70% for a 5.

- MCQ: No penalty for wrong answers. **ANSWER EVERY QUESTION.**
- FRQ: Partial credit awarded. Show ALL work. Units matter.
- Calculator sections: TI-84 or TI-Nspire CAS allowed

LIMITS & CONTINUITY

EVALUATING LIMITS

- **Direct substitution first.** If f(a) = real number, that's the limit.
- **0/0 form:** Factor, rationalize, or use L'Hopital's Rule
- **nonzero/0:** Limit is +inf, -inf, or DNE (check signs)
- **Limits at infinity:** Divide every term by highest power of x in denominator

LIMITS AT INFINITY (RATIONAL FUNCTIONS)

- deg(num) < deg(den): limit = **0**
- deg(num) = deg(den): limit = **ratio of leading coefficients**
- deg(num) > deg(den): limit = **+/- infinity (DNE)**

SPECIAL LIMITS

$\lim_{x\to 0} \sin(x)/x = 1$

$\lim_{x\to 0} (1 - \cos(x))/x = 0$

$\lim_{x\to 0} (e^x - 1)/x = 1$

L'HOPITAL'S RULE

If lim f(x)/g(x) = 0/0 or inf/inf, then lim f(x)/g(x) = lim f'(x)/g'(x). Can repeat if still indeterminate. **Only for 0/0 or inf/inf forms.**

SQUEEZE THEOREM

- If g(x) <= f(x) <= h(x) near a, and lim g(x) = lim h(x) = L, then lim f(x) = L

CONTINUITY

- f is continuous at x = a if: (1) f(a) exists, (2) lim(x->a) f(x) exists, (3) lim(x->a) f(x) = f(a)
- **Types of discontinuity:** Removable (hole), Jump, Infinite (vertical asymptote)

ALL DERIVATIVE RULES

BASIC RULES

Constant: $\frac{d}{dx}[c] = 0$

Power: $\frac{d}{dx}[x^n] = n x^{n-1}$

Constant Multiple: $\frac{d}{dx}[c f(x)] = c f'(x)$

Sum/Diff: $\frac{d}{dx}[f \pm g] = f'(x) \pm g'(x)$

Product: $\frac{d}{dx}[f g] = f'g + fg'$

Quotient: $\frac{d}{dx}[f/g] = (f'g - fg')/g^2$ ("lo d-hi minus hi d-lo over lo-lo")

Chain: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

TRIG DERIVATIVES

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

Memory trick: All "co-" functions (cos, cot, csc) have a **negative** derivative.

INVERSE TRIG DERIVATIVES

$\frac{d}{dx}[\arcsin x] = 1/\sqrt{1 - x^2}$

$\frac{d}{dx}[\arccos x] = -1/\sqrt{1 - x^2}$

$\frac{d}{dx}[\arctan x] = 1/(1 + x^2)$

EXPONENTIAL & LOGARITHMIC

$\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[a^x] = a^x \ln(a)$

$\frac{d}{dx}[\ln x] = 1/x$

$\frac{d}{dx}[\log_a(x)] = 1/(x \ln a)$

$\frac{d}{dx}[\ln|u|] = u'/u$ (chain rule form - very common on exam)

IMPLICIT DIFFERENTIATION

- Differentiate both sides with respect to x. Every time you differentiate a y-term, multiply by **dy/dx**.
- Solve for dy/dx algebraically. Factor dy/dx out if needed.

ALL INTEGRAL RULES & ANTIDERIVATIVES

BASIC ANTIDERIVATIVES

$\int x^n\,dx = x^{n+1}/(n+1) + C$ (n != -1)

$\int 1/x\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$

$\int a^x\,dx = a^x/\ln(a) + C$

$\int k\,dx = kx + C$

TRIG ANTIDERIVATIVES

$\int \cos x\,dx = \sin x + C$

$\int \sin x\,dx = -\cos x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

INVERSE TRIG ANTIDERIVATIVES

$\int 1/\sqrt{a^2 - x^2}\,dx = \arcsin(x/a) + C$

$\int 1/(a^2 + x^2)\,dx = (1/a)\arctan(x/a) + C$

U-SUBSTITUTION

Pick u = inner function. Find du. Replace ALL x-terms. For definite integrals: **change the bounds** to u-values OR convert back to x before evaluating.

PROPERTIES OF DEFINITE INTEGRALS

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^a f(x)dx = 0$

$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

$\int_a^b [c f(x)]dx = c \int_a^b f(x)dx$

FUNDAMENTAL THEOREM OF CALCULUS

FTC PART 1 (EVALUATION)

If F'(x) = f(x), then: **$\int_a^b f(x)dx = F(b) - F(a)$**

- Find the antiderivative F(x), then evaluate at the upper bound minus the lower bound.

FTC PART 2 (DERIVATIVE OF INTEGRAL)

$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$

$\frac{d}{dx}\left[\int_a^{g(x)} f(t)dt\right] = f(g(x)) \cdot g'(x)$ (chain rule version)

- **Exam favorite:** If the lower bound is also a function, split the integral at a constant and apply to each part.

KEY THEOREMS: IVT, EVT, MVT

INTERMEDIATE VALUE THEOREM (IVT)

- **If:** f is continuous on [a, b] and N is between f(a) and f(b)
- **Then:** There exists at least one c in (a, b) such that f(c) = N
- **Use:** Proving a root exists. "Since f is continuous, f(a) < 0, f(b) > 0, by IVT there exists c where f(c) = 0."

EXTREME VALUE THEOREM (EVT)

- **If:** f is continuous on **closed interval** [a, b]
- **Then:** f attains an absolute max and min on [a, b]
- **How to find:** Evaluate f at all critical points in (a, b) AND at endpoints a, b. Largest = abs max, smallest = abs min.

MEAN VALUE THEOREM (MVT)

- **If:** f is continuous on [a, b] and differentiable on (a, b)
- **Then:** There exists c in (a, b) where **f'(c) = [f(b) - f(a)] / (b - a)**
- **Meaning:** At some point, the instantaneous rate equals the average rate.

MVT FOR INTEGRALS (AVERAGE VALUE)

$f_{avg} = (1/(b-a)) \int_a^b f(x)dx$ — The average value of f on [a, b].

APPLICATIONS OF DERIVATIVES

CRITICAL POINTS & FIRST DERIVATIVE TEST

- Critical points: where f'(x) = 0 or f'(x) is undefined (and f(x) exists)
- f' changes + to - at c: **local max**
- f' changes - to + at c: **local min**
- f' > 0: f is increasing. f' < 0: f is decreasing.

SECOND DERIVATIVE TEST

- If f'(c) = 0 and f''(c) > 0: **local min** (concave up)
- If f'(c) = 0 and f''(c) < 0: **local max** (concave down)
- If f''(c) = 0: test is **inconclusive**, use first derivative test
- f'' > 0: concave up. f'' < 0: concave down.
- **Inflection point:** where f'' changes sign (concavity changes)

RELATED RATES

- 1. Draw a picture. Label ALL variables.
- 2. Write an equation relating the variables.
- 3. Differentiate both sides with respect to **t** (time). Use chain rule.
- 4. Plug in known values. Solve for the unknown rate.
- **Common setups:** Pythagorean theorem, similar triangles, $V = (1/3)\pi r^2 h$ (cone), $A = \pi r^2$ (circle)

OPTIMIZATION

- 1. Write the function to maximize/minimize.
- 2. Use a constraint to write in terms of ONE variable.
- 3. Take derivative, set = 0, solve for critical points.
- 4. Verify max/min using first or second derivative test. Check endpoints if closed interval.

LINEARIZATION & TANGENT LINE APPROX

$L(x) = f(a) + f'(a)(x - a)$

- Use the tangent line at a known point to approximate nearby values.
- If f'' > 0 (concave up): linearization is an **underestimate**
- If f'' < 0 (concave down): linearization is an **overestimate**

APPLICATIONS OF INTEGRALS

AREA BETWEEN CURVES

$A = \int_a^b |f(x) - g(x)|\,dx = \int_a^b [\text{top} - \text{bottom}]\,dx$

$A = \int_c^d [\text{right} - \text{left}]\,dy$ (horizontal slices)

- Find intersection points to determine bounds.
- If curves cross: split into separate integrals at crossing points.

VOLUMES OF REVOLUTION

Disk (no hole): $V = \pi \int_a^b [R(x)]^2\,dx$

Washer (with hole): $V = \pi \int_a^b \{[R(x)]^2 - [r(x)]^2\}\,dx$

- R(x) = distance from axis to OUTER curve. r(x) = distance from axis to INNER curve.
- **Revolving around y = k:** Adjust radius as distance from curve to that line.
- **Revolving around x = k:** Use dy integrals or shell method.

VOLUMES WITH KNOWN CROSS-SECTIONS

$V = \int_a^b A(x)\,dx$ where A(x) is the cross-section area

Shape	A(x) using side s
Square	s^2
Equilateral Triangle	$(\sqrt{3}/4) \cdot s^2$
Semicircle	$(\pi/8) \cdot s^2$
Isosceles Right Triangle	$(1/2) \cdot s^2$

- s = distance between curves (top - bottom or right - left)

ACCUMULATION FUNCTIONS

$F(x) = \int_a^x f(t)dt$ represents the accumulated quantity from a to x. F'(x) = f(x) by FTC Part 2. F is increasing when f > 0, decreasing when f < 0.

DIFFERENTIAL EQUATIONS & SLOPE FIELDS

SEPARABLE DIFFERENTIAL EQUATIONS

- 1. Rewrite dy/dx = f(x)*g(y) as **dy/g(y) = f(x)dx**
- 2. Integrate both sides. **Don't forget +C!**
- 3. Use initial condition to solve for C.
- 4. Solve for y if possible.

EXPONENTIAL GROWTH & DECAY

$dy/dt = ky \rightarrow y = y_0 e^{kt}$

- k > 0: growth. k < 0: decay.
- y_0 = initial value at t = 0.

SLOPE FIELDS

- Each segment shows the slope dy/dx at that point.
- To match: plug in (x, y) pairs into dy/dx and check if segments match.
- Solution curves follow the direction of the segments.
- Horizontal segments: dy/dx = 0 (nullclines).

EULER'S METHOD

$y_{new} = y_{old} + (\text{step size}) \cdot dy/dx|_{(x_{old},\, y_{old})}$

- $x_{new} = x_{old}$ + step size. Repeat for each step.

CALCULATOR TIPS (TI-84)

4 THINGS YOUR CALCULATOR MUST DO

- **1. Graph a function:** Y= to enter, ZOOM 6 for standard window, ZOOM FIT
- **2. Find zeros/intersections:** 2nd CALC > zero or intersect (set left bound, right bound, guess)
- **3. Numerical derivative:** MATH 8: nDeriv(f(x), x, value). Also: 2nd CALC > dy/dx on graph.
- **4. Numerical integral:** MATH 9: fnInt(f(x), x, a, b). Also: 2nd CALC > integral on graph.

COMMON CALCULATOR MISTAKES

- **Rounding too early:** Store intermediate values. Use at least 3 decimal places in FRQ answers.
- **Wrong mode:** Must be in **RADIAN** mode (not degree!) for all trig on AP exam.
- **Parentheses errors:** Always use parentheses around numerators, denominators, and exponents.
- **Forgetting window:** Graph may look empty if window is too small/large. Use ZOOM FIT.
- **Using nDeriv as proof:** Calculator gives numerical approx, NOT symbolic proof. Always show algebraic work.

FRQ STRATEGY & POINT-EARNING TIPS

6 FRQ TYPES (TYPICAL PATTERN)

#	Topic	Calc?
1	Rate/Accumulation (table or graph)	Yes
2	Particle motion or related rates	Yes
3	Graph of f' analysis	No
4	Accumulation / FTC with table	No
5	Differential equation	No
6	Area/Volume	No

HOW TO EARN MAXIMUM POINTS

- **Show all setup:** Write the integral or equation BEFORE evaluating. This earns separate points.
- **Justify with calculus:** "f has a max at x = 3 because f'(3) = 0 and f' changes from + to -."
- **Units!** Include units in every answer that has context (ft/sec, gallons, people/hour, etc.).
- **Decimal answers:** 3 decimal places or exact. Never round to 1-2 places.
- **Don't simplify:** You won't lose points for not simplifying. Don't waste time.
- **Cross out errors:** Don't erase. Cross out neatly. Graders score what's NOT crossed out.
- **Name the theorem:** "By MVT, since f is continuous on [a,b] and differentiable on (a,b)..."
- **Read every sub-part:** Parts (a)-(d) may be independent. You CAN answer (c) even if you missed (b).
- **If stuck on calculator FRQ:** Write the integral setup. Setup alone is usually 1-2 points.

MOST-TESTED TOPICS & COMMON MISTAKES

HIGHEST-FREQUENCY EXAM TOPICS

- ★★★ FTC Part 2 (derivative of integral with variable bound)
- ★★★ Interpreting f, f', f'' relationships from graphs
- ★★★ u-Substitution (definite and indefinite)
- ★★★ Area between curves / volume of revolution
- ★★★ Separable differential equations
- ★★ Related rates (Pythagorean, cone, expanding circle)
- ★★ Particle motion (position, velocity, acceleration)
- ★★ Average value of a function
- ★★ Riemann sums (left, right, midpoint, trapezoidal)
- ★★ Table problems using MVT or Riemann sums

TOP 10 COMMON MISTAKES

- **1.** Forgetting +C on indefinite integrals
- **2.** Chain rule errors (forgetting inner derivative)
- **3.** Wrong sign on trig derivatives (especially cos -> -sin)
- **4.** Not changing bounds in u-substitution for definite integrals
- **5.** Confusing position, velocity, acceleration relationships
- **6.** Using degree mode instead of radian mode on calculator
- **7.** Not checking hypotheses before applying IVT/MVT (must state continuity/differentiability!)
- **8.** Forgetting to include units in contextual FRQ answers
- **9.** Confusing f, f', f'' when reading graphs (the graph shown may be f', not f)
- **10.** Setting the derivative equal to zero but forgetting to check where f' is undefined

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Must-Memorize Formulas | Particle Motion | Riemann Sums | Complete Reference

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MUST-MEMORIZE FORMULA REFERENCE TABLE

Name	Formula
Power Rule (deriv)	d/dx [x^n] = n*x^(n-1)
Power Rule (integral)	∫ x^n dx = x^(n+1)/(n+1) + C, n!=-1
Product Rule	(fg)' = f'g + fg'
Quotient Rule	(f/g)' = (f'g - fg') / g^2
Chain Rule	[f(g(x))]' = f'(g(x)) * g'(x)
d/dx [sin x]	cos x
d/dx [cos x]	-sin x
d/dx [tan x]	sec^2 x
d/dx [e^x]	e^x
d/dx [ln x]	1/x
d/dx [a^x]	a^x * ln(a)
d/dx [arcsin x]	1/sqrt(1-x^2)
d/dx [arctan x]	1/(1+x^2)
∫ 1/x dx	ln|x| + C
∫ e^x dx	e^x + C
∫ cos x dx	sin x + C
∫ sin x dx	-cos x + C
∫ sec^2 x dx	tan x + C
FTC Part 1	∫(a to b) f(x)dx = F(b) - F(a)
FTC Part 2	d/dx [∫(a to x) f(t)dt] = f(x)
MVT	f'(c) = [f(b)-f(a)]/(b-a)
Average Value	f_avg = (1/(b-a)) ∫(a to b) f(x)dx
Disk Volume	V = pi ∫ [R(x)]^2 dx
Washer Volume	V = pi ∫ {[R]^2 - [r]^2} dx
Cross-Section Volume	V = ∫ A(x) dx
Linearization	L(x) = f(a) + f'(a)(x-a)
Euler's Method	y_(n+1) = y_n + h*f'(x_n, y_n)
Exp Growth/Decay	dy/dt = ky -> y = y_0 * e^(kt)
Trapezoidal Sum	(h/2)[f(x_0)+2f(x_1)+...+2f(x_(n-1))+f(x_n)]

PARTICLE MOTION (POSITION, VELOCITY, ACCELERATION)

RELATIONSHIPS

Position: s(t) → Velocity: v(t) = s'(t) → Acceleration: a(t) = v'(t) = s''(t)

Going backward: s(t) = ∫ v(t)dt | v(t) = ∫ a(t)dt

KEY CONCEPTS

- **Speed** = |v(t)| (absolute value of velocity, always positive)
- **Particle moves right/up** when v(t) > 0; **left/down** when v(t) < 0
- **Particle changes direction** when v(t) changes sign (v=0 and sign changes)
- **Speeding up:** v(t) and a(t) have the **same sign**
- **Slowing down:** v(t) and a(t) have **opposite signs**

DISPLACEMENT VS. TOTAL DISTANCE

Displacement: ∫(a to b) v(t) dt = s(b) - s(a) (net change, can be negative)

Total Distance: ∫(a to b) |v(t)| dt (always positive, split at sign changes)

> Exam loves asking displacement vs. total distance. Read carefully which one they want.

RIEMANN SUMS & NUMERICAL INTEGRATION

TYPES OF RIEMANN SUMS

- **Left Riemann Sum:** Use left endpoint of each subinterval: Σ f(x_i) * Delta(x)
- **Right Riemann Sum:** Use right endpoint: Σ f(x_(i+1)) * Delta(x)
- **Midpoint:** Use midpoint of each subinterval

TRAPEZOIDAL RULE

(Delta x / 2) * [f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_(n-1)) + f(x_n)]

OVER/UNDER ESTIMATES

	f is Increasing	f is Decreasing
Left Sum	Underestimate	Overestimate
Right Sum	Overestimate	Underestimate
Trapezoidal	Over if concave up, under if concave down	

> For unequal subintervals (table problems), you CANNOT use the shortcut formula. Compute each rectangle/trapezoid individually.

CONNECTING F, F', AND F'' (GRAPH ANALYSIS)

f'(x)	Means for f(x)
f'(x) > 0	f is increasing
f'(x) < 0	f is decreasing
f'(x) = 0 or DNE	Possible extremum (critical point)
f' changes + to -	f has a local **maximum**
f' changes - to +	f has a local **minimum**

f''(x)	Means for f(x)
f''(x) > 0	f is concave up (cup shape)
f''(x) < 0	f is concave down (cap shape)
f'' changes sign	f has an **inflection point**

> When they give you a **graph of f'** and ask about f: x-intercepts of f' = critical points of f. Positive area under f' = f increasing. Extrema of f' = inflection points of f.

NET CHANGE THEOREM & ACCUMULATION

> **∫(a to b) f'(x) dx = f(b) - f(a)** — The integral of a rate of change gives the net change.

- If R(t) is the rate water enters a tank: ∫(0 to 5) R(t)dt = total water added from t=0 to t=5
- If R(t) enters and D(t) drains: Amount at time T = Initial + ∫(0 to T) [R(t) - D(t)] dt
- **Exam phrasing:** "The integral represents the total number of [units] from t=a to t=b."

WHEN TO USE WHICH INTERPRETATION

- "How much total?" → ∫ |rate| dt (total distance/total amount)
- "What is the net change?" → ∫ rate dt (can be negative)
- "What is the value at time T?" → Initial value + ∫(0 to T) rate dt

RELATED RATES: COMMON FORMULAS

Pythagorean: x^2 + y^2 = z^2 → 2x(dx/dt) + 2y(dy/dt) = 2z(dz/dt)

Circle Area: A = pi*r^2 → dA/dt = 2*pi*r*(dr/dt)

Sphere Volume: V = (4/3)pi*r^3 → dV/dt = 4*pi*r^2*(dr/dt)

Cone Volume: V = (1/3)pi*r^2*h (use similar triangles to relate r and h!)

Cylinder Volume: V = pi*r^2*h

Distance: D = sqrt((x2-x1)^2 + (y2-y1)^2)

> Always ask: What is changing? What rate do I know? What rate do I need? What equation connects them?

MUST-KNOW TRIG VALUES

θ	sinθ	cosθ	tanθ
0	0	1	0
pi/6	1/2	sqrt(3)/2	1/sqrt(3)
pi/4	sqrt(2)/2	sqrt(2)/2	1
pi/3	sqrt(3)/2	1/2	sqrt(3)
pi/2	1	0	undef
pi	0	-1	0

> **Key identities:** sin^2(x)+cos^2(x)=1, 1+tan^2(x)=sec^2(x), 1+cot^2(x)=csc^2(x)

ASYMPTOTES & END BEHAVIOR

VERTICAL ASYMPTOTES

- Where the denominator = 0 BUT the numerator != 0 at that point
- If both = 0: it's a removable discontinuity (hole), NOT an asymptote

HORIZONTAL ASYMPTOTES

- Find lim(x -> +/-inf) f(x). Compare degrees of numerator and denominator.
- Can have 0, 1, or 2 horizontal asymptotes (different limits at +inf and -inf)

OPTIMIZATION: COMMON SETUPS

- **Fencing/Perimeter:** P = 2l + 2w (or 2l + w for 3-sided). A = lw. Maximize A.
- **Box with open top:** V = x^2*h, SA = x^2 + 4xh. Given SA, maximize V.
- **Minimize distance:** Minimize D^2 instead of D (avoids square root). Same answer.
- **Revenue/Profit:** R(x) = x * p(x). Profit = R(x) - C(x). Max profit: R'(x) = C'(x).

> On closed intervals: always check critical points AND endpoints. Absolute extrema require this.

LAST-MINUTE EXAM TIPS

- **Time management:** MCQ ~2 min each, FRQ ~15 min each. Skip and return.
- **MCQ strategy:** Eliminate wrong answers. Plug in answer choices. Try simple values (0, 1, -1).
- **Read carefully:** "Increasing" vs "positive." "Speed" vs "velocity." "At" vs "from."
- **Justify:** On FRQs, always explain WHY using calculus reasoning, not just what.
- **Label everything:** Variable names, units, bounds, dx or dy.
- **+C:** Include on every indefinite integral. Lose a point every time you forget.
- **Theorems:** If asked to justify, name the theorem AND verify its conditions.
- **Check your mode:** RADIAN. Every time you pick up the calculator.
- **Answers from calculator:** Write what you typed: "fnInt(2x*sin(x),x,0,3) = 5.168"
- **Don't panic on hard FRQs:** Everyone finds them hard. Partial credit adds up fast.

QUICK REFERENCE: DERIVATIVE-INTEGRAL PAIRS

f(x)	f'(x)	∫ f(x) dx
x^n	n*x^(n-1)	x^(n+1)/(n+1)+C
e^x	e^x	e^x+C
ln x	1/x	x*ln(x)-x+C
sin x	cos x	-cos x+C
cos x	-sin x	sin x+C
tan x	sec^2 x	-ln|cos x|+C
e^(kx)	k*e^(kx)	(1/k)*e^(kx)+C
1/x	-1/x^2	ln|x|+C
sqrt(x)	1/(2*sqrt(x))	(2/3)x^(3/2)+C